# Holographic Principle and Quantum Cosmology

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Using the holographic entropy proposal for a closed universe by Verlinde, a bound on equations of state for different stages of the universe is obtained. Further exploring this bound, we find that an inflationary universe naturally emerges in the early universe and today's dark energy is also needed in the quantum cosmological scenario.

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Hot big bang model says that our universe was born at some moment $t = 0$ about 13.7 billion years ago [1], in a state with infinitely large energy density and temperature. With the rapid expansion of the universe the average energy of particles and the temperature of the universe decreased rapidly and the universe became cold. This theory have been popularly accepted after the discovery of the cosmic microwave background radiation. However, like its counterpart in particle physics, Hot big bang model is not without its shortcomings, including many intrinsic difficulties, such as flatness problem, horizon problem, monopoles problem and so on. They are not inconsistencies within this model itself; rather, they involve questions that this model in the splendor of its success allows one to ask, but for which the model has yet to provide answers.

Fortunately, all these problems can be solved simultaneously in the inflationary universe scenario [2]. The idea of inflation is that the expansion of the universe during some period of the early universe, known as the inflationary stage, is accelerating. This rapid expansion made the density of monopoles vanishingly small and the size of the observable universe smaller than the Hubble size during inflation as well. Usually inflation was driven by the effective potential of the inflaton field. Nevertheless the quantum fluctuations of the inflaton field provided the seeds for the formation of the large scale structure of our universe and the temperature fluctuations in CMB, which have been confirmed by the cosmological observations, for instance [1]. But there is not a fundamental theory which predicts that inflation must happen in the early universe.

What is the initial condition for our universe is one of the deepest questions in modern physics. We believe a well-understood quantum theory of gravity is needed before we can answer this question. In the last ten years, many unexpected lessons about the nature of spacetime have been learned by string theory and black hole theory. We believe that the concept of holography must be one of the key concepts for quantum gravity [3-5]. Several conjectures of the holographic principle for cosmology have also been suggested and many consequences are obtained, for example [6-10]. In [6,7], the authors found a holographic bound on the equation of state. But the holographic principle in [6,7] fails to describe a closed univere. The covariant entropy bound proposed by Bousso in [8] is still valid for a closed universe, but the second law of thermodynamics cannot be responsible. On the other hand, there are a number of conjectures about the origin of our universe (see [11] for a brief review). As the most attractive idea among them, Hartle-Hawking no-boundary wave function $\Psi_{HH}$ of the universe [12] says that our universe was born of a tunnelling from nothing. Nothing means a state without any classical spacetime, or, a state with zero

entropy [13]. We expect that holographic principle and quantum cosmology can provide some insights on the initial conditions of our universe.

Relying on the holographic description for a closed universe proposed by Verlinde [14] and the second law of thermodynamics, we get a bound on the equation of state and we find that an inflationary universe naturally emerges and today's dark energy is also needed in quantum cosmological scenario.

Let us start with Friedman-Robertson-Walker (FRW) metric for a $(n+1)$-dimensional closed universe

$$ds^2 = -dt^2 + a^2(t)d\Omega_n^2, \tag{1}$$

where $a(t)$ represents the radius of the universe and $d\Omega_n^2$ is a short hand notation for the metric on the unit $n$-sphere $S^n$. The spatial volume of this $(n+1)$-dimensional closed FRW universe is given by

$$V = \mathrm{Vol}(S^n)a^n. \tag{2}$$

The volume of a closed universe is finite. In $n+1$ dimensional spacetime the FRW equations are given by

$$H^2 = \frac{16\pi G}{n(n-1)}\rho - \frac{1}{a^2}, \tag{3}$$

$$\dot{H} = -\frac{8\pi G}{n-1}(\rho + p) + \frac{1}{a^2}, \tag{4}$$

where $H = \dot{a}/a$ is the Hubble parameter and the dot denotes as differentiation with respect to the time $t$. In $(n+1)$ dimensions the equation of state for radiation is $w = p/\rho = 1/n$, for dust-like matter $w = 0$ and for a cosmological constant $w = -1$. Using eq. (3) and (4), we find

$$\frac{\ddot{a}}{a} = H^2 + \dot{H} = \frac{8\pi G}{n-1}\left(\frac{2}{n} - 1 - w\right)\rho. \tag{5}$$

The expansion of the universe is accelerating, if

$$w < w_c = -1 + \frac{2}{n}. \tag{6}$$

This result is also valid for a flat or an open universe.

On the other hand, the holographic principle says that the maximum entropy in a region of space is [3]

$$S_{max} = \frac{A}{4G}, \tag{7}$$

where $A$ is the area of the boundary of the region and $G$ is the Newton coupling constant. But since the space for a closed universe has no boundary, the holographic principle in its naive form (7) does not work in a closed universe.

Fortunately, Verlinde found a deep relationship between the entropy formulas for the CFT and the FRW equations for a closed universe. In [14], Verlinde focus on the case with a radiation dominated closed universe. But he also pointed out that the matching of the FRW equations and the Cardy formula is independent on the equation of state of the matter. Therefore, from the dual CFT point of view, we propose that the entropy for a closed universe whose evolution is dominated by the matter with arbitrary equation of state always takes the form

$$S = (n-1)\frac{HV}{4G}. \tag{8}$$

With the evolution of the universe, its entropy also varies. Based on the spirit of the second law of thermodynamics, it is interesting for us to investigate the variation of the entropy with time,

$$\frac{dS}{dt} = \frac{S}{H}(nH^2 + \dot{H}) = \frac{8\pi G}{n-1}\frac{S}{H}(\rho - p + 2\frac{n-1}{n}\rho_k), \tag{9}$$

where the energy density of the curvature $\rho_k$ is defined as

$$\rho_k = -\frac{n(n-1)}{16\pi G}\frac{1}{a^2}. \tag{10}$$

The energy density of the curvature is negative for a closed universe, positive for an open universe and equals zero for a flat universe. The equation of state for the curvature energy density is $w_k = -1 + 2/n$ which is just the borderline between decelerated and accelerated expansion. Physically the entropy can not be negative, because it is proportional to $\ln \mathcal{N}$, where $\mathcal{N}$ is the number of states for a system and should be a positive integer. An interesting result comes from the second law of the thermodynamics which requires that the entropy of the universe can not decrease, namely $dS/dt \geq 0$. Thus

$$\rho - p + 2\frac{n-1}{n}\rho_k \geq 0, \tag{11}$$

for an expansive universe, where the Hubble parameter $H$ is positive. We re-write the Friedman equation (3) as

$$\rho_c = \rho + \rho_k, \tag{12}$$

or equivalently, $1 = \Omega + \Omega_k$, here

$$\rho_c = \frac{n(n-1)H^2}{16\pi G} \tag{13}$$

is the critical energy density, $\Omega = \rho/\rho_c$ and $\Omega_k = \rho_k/\rho_c$. Combining eq. (11) and (12), an upper bound on the equation of state for the energy density $\rho$ is obtained

$$w = \frac{p}{\rho} \leq 1 + 2\left(1 - \frac{1}{n}\right)\frac{1-\Omega}{\Omega}. \tag{14}$$

For a flat universe, $\Omega_k = 0$, or $\Omega = 1$, eq. (14) becomes $w \leq 1$, which is just the dominant energy condition and can be interpreted as saying that the speed of energy flow of matter is always less than the speed of light. This offers an evidence to support our proposal. A similar result for a flat universe is also obtained in [6-8]. However the holographic principle for the universe proposed in [6,7] is violated for a closed universe. Even though the covariant entropy bound proposed by Bousso in [8] is still valid for a closed universe, it is time reversal invariant and the second law of thermodynamics cannot be responsible. Here we stress that the starting point of ours is quite different from those by the authors of [6-9] and the second law of thermodynamics leads to a bound on the equation of state as eq. (14).

Quantum cosmology is an elegant idea to probe the origin of our universe, which says that our universe was born of a tunnelling from nothing. Since the volume of the spatial flat or open universe is infinite, unless the topology or the configuration of the universe is nontrivial [15], the tunneling probability from nothing to any one of them is suppressed. Here we focus on the case with trivial topology and only a closed universe can emerge. In quantum cosmology scenario, the initial state corresponds to $H = \dot{a}/a = 0$. Nothing is interpreted as the initial state of our universe with zero entropy [13], since the entropy of a closed universe takes the form as (8).

The tunnelling probability corresponding to Hartle-Hawking wave function of the universe is given by $\mathcal{P}_{HH} \simeq \exp\left(\frac{3}{8\Lambda}\right)$. A universe with cosmological constant $\Lambda = 0$ is favored. This universe is empty. It contradicts the Hot big bang history of our universe. We expect that the effects of quantum gravity can improve the wave function of the universe to prefer a universe with matter within it when it was born, for instance [16-20,13]. Since the Hubble parameter equals zero when the universe was created, the critical energy density equals zero by using eq. (13). Now $\Omega = -\Omega_k \to +\infty$. Using eq. (14), Holographic

principle for a closed universe requires the equation of state for the matter in the initial state of our universe satisfy

$$w_i \leq -1 + \frac{2}{n}. \tag{15}$$

Thus a universe filled with radiation or the dust-like matter can not be created in the quantum cosmology scenario. If the equation of state for the matter when the universe was born is roughly a constant, the expansion of the universe must be accelerating, or equivalently, an inflationary universe naturally emerges. On the other hand, Wheeler-DeWitt (WdW) equation which describes the evolution of the wave function of the universe $\Psi$ is

$$\left(\hat{\Pi}_a^2 + U(a)\right) \Psi = 0, \tag{16}$$

where the potential is $U(a) \sim a^{2n-4} - a^{2n-2}G\rho$, $\hat{\Pi}_a = -i\partial/\partial a$ and the energy density for the matter satisfies $\rho \sim a^{-n(w+1)}$. In order that there is a barrier in the potential $U(a)$ and the universe can be created from nothing, the equation of state for the matter should satisfy $w < -1 + 2/n$ which is consistent with our above argument and prefers an inflationary universe. A similar result in four dimensions has also been obtained in [19].

In [9], Bak and Rey suggested to consider apparent horizon instead of the particle horizon instead of the particle horizon in [6] and they found a constraint on the equation of state as $w \leq -1+2/n$ in $(n+1)$ dimensions. This constraint means that the expansion of the universe is always accelerating, which conflicts with the history of our universe. But the bound suggested by us in eq. (14) is softer. When our universe was born, $\Omega = -\Omega_k \to +\infty$ and $w \leq -1 + 2/n$. After a stage of inflation, the energy density of the curvature was inflated away and $\Omega \to 1$. Now the constraint on the equation of state became $w \leq 1$, which allows that the reheating occurred and the evolution of our universe could be dominated by radiation or matter.

On the other hand, since the geometry of a universe can not be changed classically, we can expect our universe is always closed in the quantum cosmology scenario. If there are only radiation and dust-like matter in such a closed universe, the bound on the equation of state (14) will be violated sooner or later, unless there is also dark energy with $w < -1 + 2/n$. For instance, we take a closed universe dominated by dust-like matter into account. The Friedman equation takes the form

$$H^2 = \frac{16\pi G}{n(n-1)} \frac{\rho_0}{a^n} - \frac{1}{a^2}, \tag{17}$$

with $a_0 = 1$. When the scale factor goes to

$$a = a_{max} = \left(\frac{16\pi G\rho_0}{n(n-1)}\right)^{\frac{1}{n-2}}, \tag{18}$$

Hubble parameter equals zero and the universe begins to collapse. Now the bound (14) becomes $w \leq -1 + 2/n$ and is violated. A more careful consideration tells us that the holographic bound (14) has been violated before $a \to a_{max}$. To avoid this problem, there should be a matter, named dark energy, with equation of state $w < -1+2/n$. Otherwise the universe will collapse and the holographic bound (14) is violated. Therefore we says that the holographic principle predicts that there must be dark energy today in our universe. In fact, the dark energy has also been confirmed by the cosmological observation [1,21] at a high level of statistical significance.

To summarize, we obtain a bound on the equation of state for the matter in a flat or closed universe. Using this bound, we find that an inflationary universe naturally emerges and today's dark energy is also needed in the history of our universe in the quantum cosmological scenario. The matching of the whole history of our universe seems mysterious, but it can be taken as evidence that the our proposal is on the right track. It is the first time that the accelerated expansion is necessary from the fundamental theory point of view. Since the initial entropy of our universe in the quantum cosmology equals zero, we can also expect that the entropy should increase with the classical evolution of our universe. This naturally provides an arrow of time, i.e. along the line with entropy increasing. Because Hubble parameter and the entropy drops to zero when the closed universe begins to collapse, our universe expands for ever; otherwise, the second law of the thermodynamics will be violated.

Unfortunately, we do not understand the microscopic physics about the deep correspondence between Friedman equation for a closed universe and the formulation for the entropy of the CFT. The physical meaning of the entropy formula (8) is also unknown. We believe that this deep duality encodes some unknown, but important insights on the holographic principle in a no boundary system, and it may play a critical role on our understanding of the nature of the spacetime. We also hope this work can open a window to understand the history of our universe.

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